Journal of Research in Multidisciplinary Methods and Applications
http://www.satursonpublishing.com/
ISSN: 2957-3920

Volume 1, Issue 4, August 2022

On Non-Homogeneous Ternary Cubic Diophantine Equation $w^2+2z^2-2wx-4zx=9x^3-3x^2$

S. Vidhyalakshmi[1], M.A. Gopalan[2]

[1]Assistant Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

[2]Professor, Department of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Trichy-620 002, Tamil Nadu, India.

Abstract: The non-homogeneous ternary cubic diophantine equation $w^2+2z^2-2wx-4zx=9x^3-3x^2$ is analyzed for its patterns of non-zero distinct integral solutions. A few relations between the solutions and special number patterns are presented.

Keywords: Ternary cubic Non- Homogeneous cubic, Integral solutions

1 Introduction

The Diophantine equation offers an unlimited field for research due to their variety [1-3]. In particular, one may refer [4-12] for cubic equations with three unknowns. This communication concerns with yet another interesting equation $w^2+2z^2-2wx-4zx=x^3-3x^2$ representing non-homogeneous cubic with three unknowns for determining its infinitely many non-zero integral points. A few relations between the solutions and special number patterns are presented.

2 Method Of Analysis

The given non-homogeneous ternary cubic diophantine equation is

$$w^2+2z^2-2wx-4zx=9x^3-3x^2 \tag{1}$$

To start with, it is seen that (1) is satisfied by the integer triples given below:

$$(x,z,w)=(\alpha^2,\alpha^2+2\alpha^3,\alpha^2+\alpha^3),(3\alpha^2,3\alpha^2+11\alpha^3,3\alpha^2+\alpha^3),$$
$$(6\alpha^2,6\alpha^2+30\alpha^3,6\alpha^2+12\alpha^3)$$

However, we have other sets of integer solutions to (1).We illustrate below

the process of obtaining different sets of integer solutions to (1):

Set 1:

On completing the squares,(1) is written as

$$P^2+2Q^2=9x^3 \tag{2}$$

where

$$P=w-x, Q=z-x \tag{3}$$

After some algebra,it is observed that (2) is satisfied by

$$P=3m(m^2+2n^2), Q=3n(m^2+2n^2) \tag{4}$$

and

$$x=m^2+2n^2 \tag{5}$$

From (4) and (3) ,we have

$$w=(3m+1)(m^2+2n^2), z=(3n+1)(m^2+2n^2) \tag{6}$$

Thus,(5) and (6) represent the integer solutions to (1).

Set 2:

Write 9 as

$$9=(1+i2\sqrt{2})(1-i2\sqrt{2}) \tag{7}$$

Substituting (5) and (7) in (2) and employing the method of factorization one has

$$P+i\sqrt{2}Q=(1+i2\sqrt{2})(m+i\sqrt{2}n)^3 \tag{8}$$

On equating the real and imaginary parts ,we have

$$P=(m^3-6mn^2)-4(3m^2n-2n^3),Q=2(m^3-6mn^2)+(3m^2n-2n^3) \quad (9)$$

Using (9) in (3), note that

$$z=m^2+2n^2+3m^2n-2n^3+2(m^3-6mn^2),$$
$$w=m^2+2n^2-4(3m^2n-2n^3)+(m^3-6mn^2) \quad (10)$$

Thus,(5) and (10) represent the integer solutions to (1).

Note 1:

The integer 9 on the R.H.S. of (2) is also represented by

$$9=\frac{(7+i4\sqrt{2})(7-i4\sqrt{2})}{9}$$

The repetition of the above process leads to a different set of solutions to (1).

Set 3:

Write (2) as

$$P^2+2Q^2=9x^3*1 \quad (11)$$

Consider 1 as

$$1=\frac{(1+i2\sqrt{2})(1-i2\sqrt{2})}{9} \quad (12)$$

Using (5) ,(7) and (12) in (11) and employing the method of factorization, one has

$$P+i\sqrt{2}Q=\frac{(-7+i4\sqrt{2})(m+i\sqrt{2}n)^3}{3}$$

Following the procedure as in Set 2 and replacing m by 3M,n by 3N,the integer

solutions to (1) are given by

$$x=9(M^2+2N^2),z=9(-7(M^3-6MN^{2)}-8(3M^2N-2N^3)+M^2+2N^2),$$
$$w=9(4(M^3-6MN^2)-7(3M^2N-2N^3)+M^2+2N^2)$$

Note 2:

One may also take 1 on the R.H.S. of (11) in general as

$$1=\frac{(2r^2-s^2+i2rs\sqrt{2})(2r^2-s^2-i2rs\sqrt{2})}{(2r^2+s^2)^2},$$
$$1=\frac{(7+i6\sqrt{2})(7-i6\sqrt{2})}{11^2}$$

The repetition of the above process leads to different sets of solutions to (1).

3 CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to (1). To conclude, one may search for integer solutions to other choices of homogeneous or non-homogeneous ternary cubic Diophantine equations.

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