Journal of Research in Multidisciplinary Methods and Applications
http://www.satursonpublishing.com/
ISSN 2957-3920 (Online) ISSN 3007-7060 (Print)
Volume 3, Issue 11, November 2024

Reducer Design on Hypocycloid Planetary Transmission

Zhang Qiang*, Zhu Zhenglong, Zhang Nanqing, Chen Xiaoyu

College of Engineering, Zunyi Normal University, Zunyi 563006, China

*Correspondence to: Zhang Qiang

Abstract: Theoretical analysis was conducted on the formation principle of the hypocycloid, and the tooth profile curve equation of the hypocycloid was constructed using the inner side method of the base circle. Based on the analogy of hypocycloid planetary transmission, a structural composition scheme for a pin wheel planetary reducer based on hypocycloid was designed, and three-dimensional structural models of each component of the reducer were established in SolidWorks software. The complete hypocycloid planetary reducer model was obtained through assembly.

Keywords: Hypocycloid; Planetary transmission; Retarder; Three-dimensional model

1 INTRODUCTION

Compared with ordinary involute gear transmission, cycloidal pin wheel planetary transmission has the advantages of large transmission ratio, high accuracy, good stiffness, and significant error averaging effect, and is widely used in various precision transmission fields. From the perspective of composition principles, cycloids can be divided into two types: hypocycloid and epitrochoid. In recent years, many scholars have done a lot of beneficial work in the field of cycloidal gear meshing transmission. Wang Qin[1] established an optimization design method for the meshing efficiency of cycloidal hypoid gears based on the contact analysis method of gear friction loading. Zhang Yueming[2] established a multi tooth load-bearing contact analysis model for the cycloidal pinwheel transmission mechanism considering friction and meshing backlash, and calculated the meshing force and load distribution law of the cycloidal pinwheel. Cui Jiankun[3] derived the calculation formula for the pressure angle of the internal and external cycloid gears in the double cycloid gear transmission through graphical method, and obtained the variation law of the pressure angle during the meshing process, as well as the influence of the main parameters of the double cycloid gear on the pressure angle of the internal and external cycloid gears. Fu Chao[4] constructed a multi-body dynamics model and analyzed the transmission efficiency and fatigue strength of the cycloidal gear transmission mechanism through the design of cycloidal gear tooth profile modification. Su Jianxin[5] constructed mathematical models for contact stress and meshing stiffness of cycloidal needle gear pairs, and conducted relevant force analysis and optimization. Ta-Shi Lai[6] established the meshing equation of cycloidal pinwheel transmission based on the envelope method of surface single parameter. Li Xuan[7] determined the time-varying meshing parameters and load distribution of cycloidal pinwheel gear pairs based on tooth contact analysis and nonlinear Hertz contact theory, and achieved accurate calculation of the contact stiffness of single tooth pairs and the torsional stiffness of multi tooth pairs. Wang Junzheng[8] proposed a Monte Carlo simulation based error compensation and correction model for cycloidal gear machining, providing a new theoretical method for cycloidal gear machining error compensation. Tang Weimin[9] derived and designed a cycloidal bevel gear meshing pair with a double circular arc tooth profile based on meshing theory, and established a finite element model to simulate its meshing characteristics. However, most of these studies are focused on hypocycloids, and there is little mention of meshing theory research and related reducer design for hypocycloids. Based on this, this article takes the cycloid as the research object, explores its formation method, and designs the corresponding reducer structure, laying the foundation for its subsequent market application.

2 GENERATION OF INVOLUTE PROFILE CURVE

Like the involute profile, the involute profile has two formation methods: inner base circle formation method and outer base circle formation method. The method of forming the outer side of the base circle is suitable for situations where the rolling circle is relatively large. Due to the relatively small diameter of the hypocycloid rolling circle designed in this article, the method of forming the inner side of the base circle is used for generation.

Journal of http://www.satursonpublishing.com/
Research in Multidisciplinary Methods and Applications
ISSN 2957-3920 (Online) ISSN 3007-7060 (Print)
Volume 3, Issue 11, November 2024

2.1 PRINCIPLE OF INNER BASE CIRCLE FORMATION METHOD

As shown in Figure 1, there are two circles inscribed at point P. Among them, a circle with a radius of r is called a rolling circle; A circle with a radius of R is called a base circle. When the base circle is fixed and the rolling circle rolls purely along the inner side of the base circle, the trajectory formed by the point C1 on the rolling circle is called an hypocycloid.

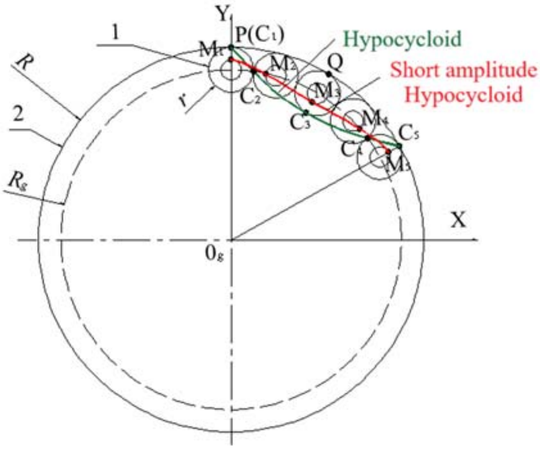


FIGURE 1: FORMATION METHOD ON INNER BASE CIRCLE

The amplitude and height of the hypocycloid are denoted as h0. As shown in Figure 1, h0=2r, which is equal to the diameter of the rolling circle; The average radius of the hypocycloid is Rg=R+r. A point M1 located within and firmly connected to the rolling circle, denoted as the distance between point M1 and the center O, is called the eccentricity, recorded as e. The trajectory curve M1M5 of point M1 is called a short amplitude hypocycloid, with a height h equal to twice the eccentricity, h=2e. Because e<r, h<h0. The ratio of the amplitude height h of the short amplitude hypocycloid to the amplitude height h0 of the normal hypocycloid is denoted as the short amplitude coefficient K1.

$$K_1 = \frac{h}{h_0} = \frac{2e}{2r} = \frac{e}{r} < 1 \qquad (1)$$

The average radius of the short amplitude epitrochoid is

$$R_g = R + \mathrm{M_1C_1} + \frac{h}{2} = R + r \qquad (2)$$

As can be seen from the above formula, the average radius of the short amplitude epitrochoid is equal to that of the ordinary epitrochoid.

2.2 CONSTRUCTION OF HYPOCYCLOID PROFILE CURVE EQUATION

According to the formation principle of the inner side method of hypocycloid, two coordinate systems are established in Figure 2: the coordinate system X1O1Y1 fixedly connected to the rolling circle and the overall fixed coordinate system XOY. Assuming the radius of the base circle is R, the radius of the rolling circle is r, the distance from point P to the center of the rolling circle is e, the angle at which the rolling circle rotates around the base circle is denoted as φ, and the angle at which the rolling circle rotates on its own is denoted as θ.

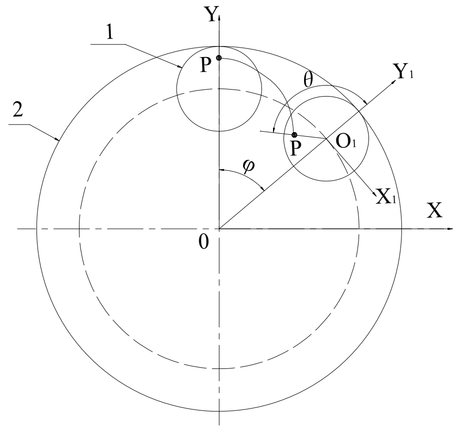


FIGURE 2: THE ESTABLISHED COORDINATE SYSTEM

By analyzing the motion process, the trajectory curve equation of point P in the X1O1Y1 coordinate system can be obtained as follows:

$$\begin{cases} x_1 = -\sin\theta \\ y_1 = \cos\theta \end{cases} \qquad (3)$$

The transformation from coordinate system X1O1Y1 to coordinate system XOY requires two transformations: translation transformation and rotation transformation. So the trajectory curve equation of point P in the XOY coordinate system can be obtained by the following equation:

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & R-r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

The equation for the trajectory curve of point P is:

$$\begin{cases} X = e\sin(\varphi-\theta) + (R-r)\sin\varphi \\ Y = e\cos(\varphi-\theta) + (R-r)\cos\varphi \end{cases} \qquad (4)$$

Among them, there is a relationship between φ and θ as follows:

$$\varphi = \frac{\theta \cdot r}{R} = \frac{\theta}{Z_g} \qquad (5)$$

In equation 5, Zg is the ratio of the base circle radius to the rolling circle radius, which is the number of hypocycloid gear. As deduced earlier, the actual tooth profile of hypocycloid gear is the equidistant line of the short amplitude hypocycloid.

Journal of http://www.satursonpublishing.com/
Research in Multidisciplinary Methods and Applications
ISSN 2957-3920 (Online) ISSN 3007-7060 (Print)
Volume 3, Issue 11, November 2024

According to the knowledge of differential geometry [10], equidistant lines are equidistant along the normal direction of the point on the original curve. Therefore, taking the derivative of equation 4 can obtain the normal equation of any point on the curve.

$$\begin{cases} \dfrac{dX}{d\theta} = \dfrac{e(r-R)}{R}\cos(\varphi-\theta) + \dfrac{(R-r)r}{R}\cos\varphi \\ \dfrac{dY}{d\theta} = -\dfrac{e(r-R)}{R}\sin(\varphi-\theta) - \dfrac{(R-r)r}{R}\sin\varphi \end{cases}$$

So the slope of the normal at any point on the curve is:

$$k = -\frac{dX}{dY} = -\frac{dX/d\theta}{dY/d\theta} = \frac{e\cos(\varphi-\theta) - r\cos\varphi}{e\sin(\varphi-\theta) - r\sin\varphi} \quad (6)$$

Assuming the inclination angle of the normal is γ, equation 6 yields:

$$\begin{cases} \cos\gamma = \pm\dfrac{e\sin(\varphi-\theta) - r\sin\varphi}{\sqrt{e^2 + r^2 - 2re\cos\theta}} \\ \sin\gamma = \pm\dfrac{e\cos(\varphi-\theta) - r\cos\varphi}{\sqrt{e^2 + r^2 - 2re\cos\theta}} \end{cases} \quad (7)$$

If the distance between the equidistant line and the epitrochoid is rz, the equidistant line equation of hypocycloid can be obtained from equations 4 and 7, that is, the actual tooth profile curve equation of hypocycloid gear is:

$$\begin{cases} X = e\sin(\varphi-\theta) + (R-r)\sin\varphi + r_Z\cos\gamma \\ Y = e\cos(\varphi-\theta) + (R-r)\cos\varphi + r_Z\sin\gamma \end{cases} \quad (8)$$

Due to the fact that the actual involute profile is generally taken as the outer contour of the theoretical profile, the "-" sign is used in Equation 7, that is:

$$\begin{cases} \cos\gamma = -\dfrac{e\sin(\varphi-\theta) - r\sin\varphi}{\sqrt{e^2 + r^2 - 2re\cos\theta}} \\ \sin\gamma = -\dfrac{e\cos(\varphi-\theta) - r\cos\varphi}{\sqrt{e^2 + r^2 - 2re\cos\theta}} \end{cases} \quad (9)$$

2.3 *EXAMPLE ANALYSIS*

To verify the correctness of the tooth profile curve formula, two sets of example values were taken for calculation, as shown in Table 1 below. The calculated gear curves are shown in Figures 3 and 4 below:

TABLE 1: EXAMPLE PARAMETER LIST

Parameters	R(mm)	r(mm)	rz(mm)	Zg	e(mm)
Example 1	100	10	10	10	5
Example 2	100	6.25	10	16	5

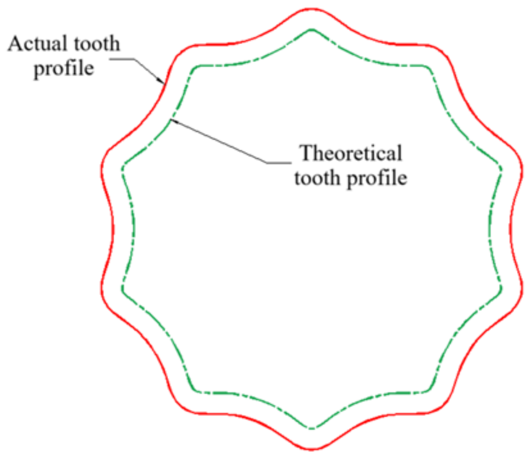


FIGURE 3: TOOTH PROFILE CURVE GENERATED IN EXAMPLE 1

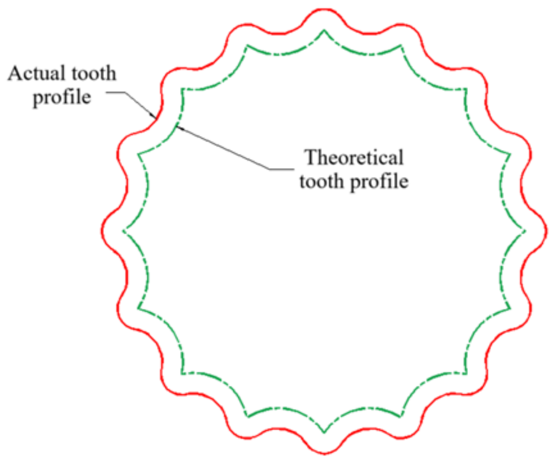


FIGURE 4: TOOTH PROFILE CURVE OBTAINED FROM EXAMPLE 2

By comparing Figures 4 and 5, it can be concluded that when other parameters are the same, the larger the number of teeth Zg, the smaller the curvature radius of the tooth profile curve. Therefore, the transmission characteristics are better during the transmission process, but the smoothness of the transmission is relatively poor; On the contrary, the smaller the number of teeth Zg, the larger the curvature radius of the tooth profile curve. Therefore, the transmission characteristics are better during the transmission process, but the smoothness of the transmission is relatively poor.

Journal of Research in Multidisciplinary Methods and Applications http://www.satursonpublishing.com/
ISSN 2957-3920 (Online) ISSN 3007-7060 (Print)
Volume 3, Issue 11, November 2024

3 STRUCTURAL DESIGN OF HYPOCYCLOID PINWHEEL PLANETARY REDUCER

The principle of hypocycloid pinwheel planetary transmission is consistent with that of epitrochoid planetary transmission. However, in hypocycloid pinwheel planetary transmission, the tooth profile of the cycloid wheel is the equidistant line of the hypocycloid wheel. Therefore, in meshing transmission, the hour wheel should be arranged on the inner side of the cycloid wheel, which belongs to internal meshing transmission. Referring to the structure of epitrochoid planetary transmission, the structure of hypocycloid pinwheel planetary transmission can be obtained as shown in Figure 5.

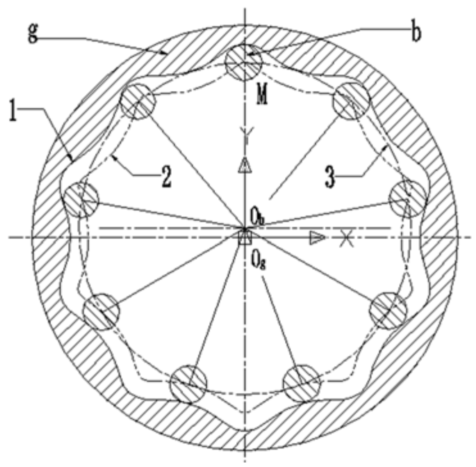


FIGURE 5: STRUCTURAL SCHEME OF CYCLOID NEEDLE WHEEL PLANETARY TRANSMISSION

In Figure 5, the needle teeth b are arranged at the intersection of a circle with radius r=ObM and the hypocycloid 2, and the actual tooth profile 1 is the equidistant line of the theoretical tooth profile 2. The cycloid gear g is fixed, and the needle teeth b are fixedly connected to the cycloid gear. The center of its distribution circle Ob revolves around the center Og of the cycloid gear.

The structure of the internal cycloid pin wheel planetary transmission mainly includes three parts: input structure, transmission structure, and output structure.

3.1 *INPUT STRUCTURE DESIGN*

In an hypocycloid planetary gear reducer, the input shaft (rotating arm H) is generally used as the active component to transmit the rotational speed to the pinion. Referring to the structure of the cycloidal pinwheel planetary transmission, the rotating arm H relies on eccentricity to drive its orbital motion, so there must be eccentricity in the structure. Due to the existence of this eccentricity, it inevitably leads to instability during shaft rotation, so a symmetrical eccentric structure design is needed to mitigate the adverse effects caused by eccentricity. At the same time, for the convenience of processing, the shaft is generally not directly made into an eccentric shaft, but an independent eccentric shaft sleeve design is adopted. Therefore, the composition of its input structure is shown in Figure 6.

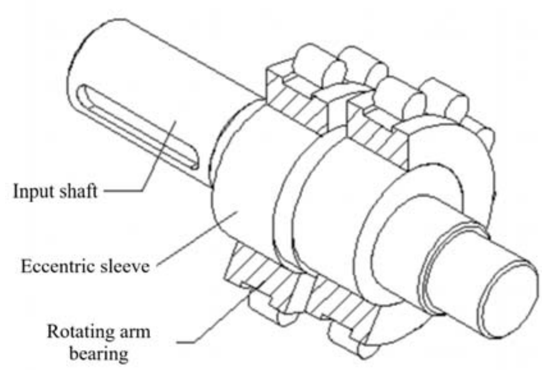


FIGURE 6: INPUT STRUCTURE

3.2 *TRANSMISSION STRUCTURE DESIGN*

In an hypocycloid planetary reducer, due to the orbital motion of the pinwheel, dynamic imbalance problems may occur during operation. To eliminate this problem, two needle wheels are usually symmetrically arranged, so that during operation, the eccentricities of the two needle wheels cancel each other out, thereby achieving smooth transmission. The structure of meshing transmission between two needle wheels and a cycloid wheel is shown in Figure 7.

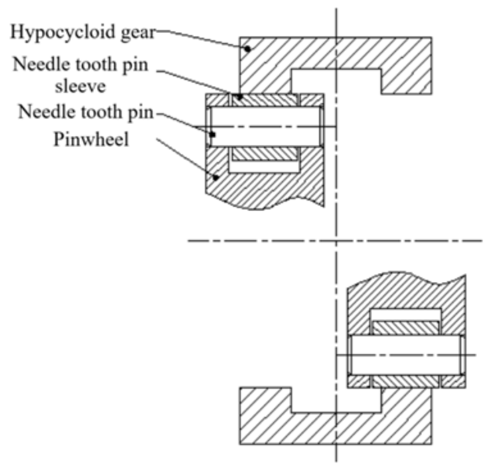


FIGURE 7: LOCAL STRUCTURE OF THE MESHING BETWEEN HYPOCYCLOID GEAR AND PINWHEEL

In Figure 7, the cycloid gears that mesh with two needle gears are merged into one internal cycloid gear, with identical tooth profiles on both sides. During the transmission process, the motion of the two needle wheels at any time is completely symmetrical, so the transmission is relatively smooth.

Journal of http://www.satursonpublishing.com/
Research in Multidisciplinary Methods and Applications
ISSN 2957-3920 (Online) ISSN 3007-7060 (Print)
Volume 3, Issue 11, November 2024

3.3 OUTPUT STRUCTURE DESIGN

The output of the hypocycloid pinwheel planetary reducer is the rotational motion of the pin wheel. In order to overcome the influence of revolution and achieve the output of rotational motion, the output structure adopts a parallelogram mechanism(W mechanism), as shown in Figure 8.

In Figure 8, the column pin is fixedly connected to the output shaft, transmitting motion to the output shaft for output. In order to form a parallelogram mechanism, the eccentricity between the pin hole and the pin shaft should be equal to the distance between the center of the pinwheel and the center of rotation of the input shaft. Therefore, the distance AOb from the center of the pin hole to the center of the pinwheel is equal to the distance BOg from the center of the pin shaft to the center of rotation of the input shaft. Thus, ABOgOb forms a parallelogram. According to mechanical principles, a parallelogram belongs to a hyperbolic handle structure with 2 degrees of freedom. Therefore, the rotation of rod AB (the revolution of the pinwheel) does not affect the rotation of rod AOb (the rotation of pinwheel), and the rotation of rod AOb is also equal to the rotation of rod BOg (the output motion of output shaft). So, the output shaft outputs the rotational motion of the needle wheel.

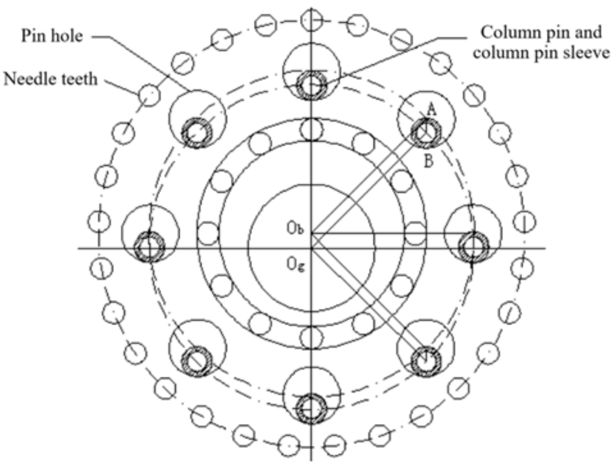


FIGURE 8: PIN HOLE W OUTPUT MECHANISM

4 STRUCTURAL MODEL OF HYPOCYCLOID PINWHEEL PLANETARY REDUCER

To verify the feasibility of the hypocycloid planetary transmission, a three-dimensional structural model of the hypocycloid pinwheel planetary reducer was designed in SolidWorks 2012 software based on the above structural scheme. The relevant components and physical models of the entire machine are shown in figures 9-16 below:

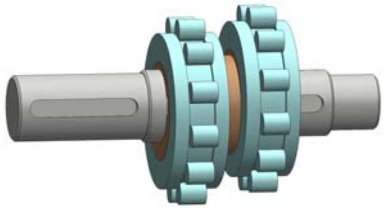


FIGURE 9: INPUT STRUCTURE

FIGURE 10: HYPOCYCLOID GEAR

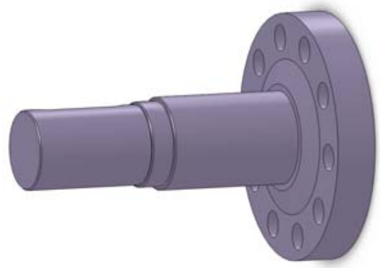
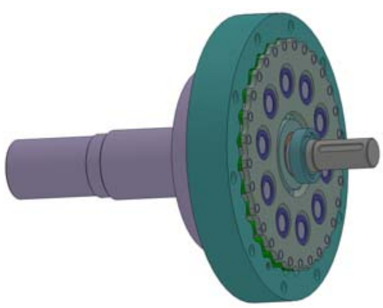


FIGURE 11: OUTPUT SHAFT

FIGURE 12: NEEDLE WHEEL



Journal of http://www.satursonpublishing.com/
Research in Multidisciplinary Methods and Applications
ISSN 2957-3920 (Online) ISSN 3007-7060 (Print)
Volume 3, Issue 11, November 2024

FIGURE 13: TRANSMISSION SYSTEM ASSEMBLY

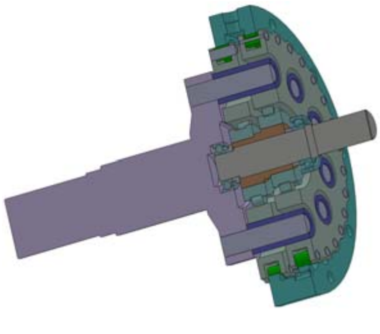


FIGURE 14 SECTIONAL VIEW

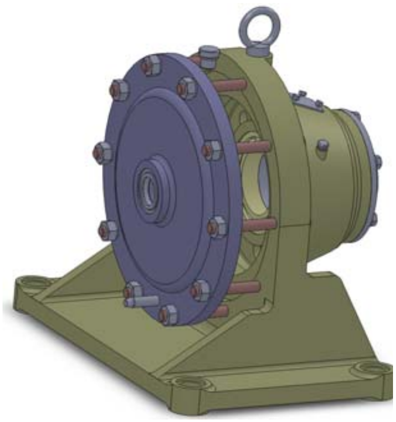


FIGURE 15: BOX STRUCTURE ASSEMBLY

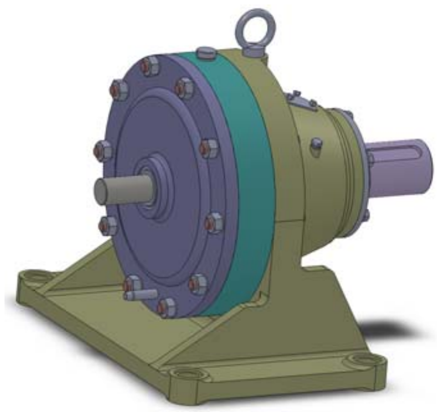


FIGURE 16: COMPLETE REDUCER MODEL

According to the designed model of the cycloidal pinwheel planetary reducer, it is known that the cycloidal can be used in the transmission field and has the characteristics of compact structure and high transmission ratio, which is suitable for applications in specific scenarios.

5 CONCLUSION

This article introduces the principle of forming the inner side of the base circle of the hypocycloid, derives the equation of the profile curve of the hypocycloid and generates the actual profile curve of the hypocycloid by combining two sets of examples. Through comparison, it is found that the larger the number of teeth, the better the force characteristics of the hypocycloid transmission process, but it will reduce the smoothness of the transmission. Finally, by analogy with the structural scheme of the cycloidal pinwheel planetary transmission, the structural composition scheme of the hypocycloid pinwheel planetary transmission was designed, and three-dimensional models of each component and the whole machine were constructed in SolidWorks software, laying a theoretical foundation for the design of the hypocycloid planetary reducer.

FUNDING

Zunhong Kehe Shizi [2022] No.06, Zunshi Kehe HZ[2023] No.163, Zunshi Kehe HZ[2023] No.152.

REFERENCE

[1]Wang Qin, He Di, Xue Jianhua, et al. Research on Optimization Design of Gear Mesh Efficiency of Drive Axle Cycloid Hypoid Gear [J] China Mechanical Engineering, 2024, 35 (07): 1-10

[2]Zhang Yueming, Li Tianyu, Ji Shuting. The Influence and Parameter Optimization of Cycloid Needle Wheel Reducer Parameters on Transmission Efficiency [J] Journal of South China University of Technology (Natural Science Edition), 2024, 52 (04): 77-87

[3]Cui Jiankun, Ding Jiale, Shi Yuheng, et al. Analysis of tooth profile and meshing characteristics of double cycloid needle gear transmission [J] Mechanical strength, 2024, 46 (02): 424-430

[4]Fu Chao, Guo Xiangkun, Gu Zhenyu, et al. Research on the Transmission Performance of Cycloid Gear actuators [J] Transmission Technology, 2024, 38 (01): 9-12

[5]Su Jianxin, Wang Dongfeng, Xu Jiake, et al. Calculation and simulation analysis of meshing stiffness of cycloidal pinwheel [J] Mechanical Transmission, 2024, 48 (07): 1-7.

[6]Ta-Shi Lai.Design and machining of the epicycloid planet gear of cycloid drives. International Journal of Advanced Manufacturing Technology,2006,28:665～670.

[7]Li Xuan, Chen Bing-kui, Wang Ya-wen, et al. Mesh stiffness calculation of cycloid-pin gear pair with tooth profile modification and eccentricity error[J].Journal of Central South University,2018,25(07):1717-1731.

[8]Junzheng W, Hongzhan L. Modification and Optimization of Cycloidal Gear Tooth Profile Based on Machining Error Compensation[J].Applied Sciences,2023,13(4):2581-2581.

[9]Weimin T, Yinkun H, Yihuang W, et al. Geometry design, meshing analysis and error influences of face-hobbed cycloidal bevel gears with double circular-arc profile for a nutation drive[J].Mechanism and Machine Theory,2022,176.

[10]Chen Weihuan. Differential Geometry (Second edition) [M]. Peking University Press, August 2017.